All of the comments for A336013 also describe this table. Remember that $I = -\frac{z}{y}$ and $\theta = -\frac{y}{x}$.
I and θ are both integers only for rows with x = +-1.

There exist pairs of rows $[x_1, y_1, z_1]$ and $[x_2, y_2, z_2]$ for which $I_2 = \theta_1$ and $\theta_2 = I_1$ making $\frac{I_1}{\theta_2} + \frac{I_2}{\theta_1} = 1 + 1 = 2$. They provide the simplest cases of summing two rows to get a triple for which $y^2 - y - xz = 0$. In these cases the sum of the two rows is always $[0, 1, 0]$ which is not in the table but corresponds to $0ab + 1(a+b) + 0 = a+b$.

Proof that when the I's and θ's are switched between $f_1(a,b)$ and $f_2(a,b)$,
$f_1(a,b) + f_2(a,b) = a+b$:

Given $I_2 = \theta_1$ and $\theta_2 = I_1$,

$$f_1(a,b) = \frac{ab - \theta_1(a+b) + I_1\theta_1}{I_1 - \theta_1}$$

$$f_2(a,b) = \frac{ab - \theta_2(a+b) + I_2\theta_2}{I_2 - \theta_2}$$

$$= \frac{ab - I_1(a+b) + \theta_1 I_1}{\theta_1 - I_1}$$

$$= \frac{-ab + I_1(a+b) - I_1\theta_1}{I_1 - \theta_1}$$

$$f_1(a,b) + f_2(a,b) = \frac{ab - ab + (I_1 - \theta_1)(a+b) + I_1\theta_1 - I_1\theta_1}{I_1 - \theta_1} = a+b \quad \text{QED.}$$

With the introduction of negative x, y and z we have the possibility of summing three rows to a triple for which $y^2 - y - xz = 0$, and each pair of the three rows sums to a row. This was not possible in A336013. However when this happens, the result is not a row. This is because in these cases, the sum of three rows is always $[0, 1, 0]$.

Proof that if three rows pairwise sum to another row, then all three sum to $[0, 1, 0]$:

<u>Lemma</u>. If $[x, y, z]$ is a row, then $[0, 1, 0] - [x, y, z]$ is a row.

Proof of lemma. $[0, 1, 0] - [x, y, z] = [-x, 1-y, -z]$.
Rename this triple $[X, Y, Z]$ and show that $Y^2 - Y - XZ = 0$.
$Y^2 - Y - XZ = (1-y)^2 - (1-y) - (-x)(-z) = 1 - 2y + y^2 - 1 + y - xz$
$= y^2 - y - xz = 0$.

<u>Corollary</u>. If $[x, y, z]$ is not a row, then $[0, 1, 0] - [x, y, z]$ is not a row.

Given rows $[x_1, y_1, z_1]$ and $[x_2, y_2, z_2]$ that sum to another row.
By the lemma, $[0, 1, 0] - ([x_1, y_1, z_1] + [x_2, y_2, z_2])$ equals a row, call it $[x_3, y_3, z_3]$.
Rearranging the equation
so that $[x_1, y_1, z_1]$ alone is on the right side we see that $[x_2, y_2, z_2] + [x_3, y_3, z_3]$ is a row;
so that $[x_2, y_2, z_2]$ alone is on the right side we see that $[x_1, y_1, z_1] + [x_3, y_3, z_3]$ is a row.
Moving all the triples to the right side we get
$[0, 1, 0] = [x_1, y_1, z_1] + [x_2, y_2, z_2] + [x_3, y_3, z_3]$ QED.

Examples of two rows that sum to $[0, 1, 0]$ and I's and θ's are switched:

(1) $[x_1, y_1, z_1] + [x_2, y_2, z_2] = [2, 2, 1] + [-2, -1, -1] = [0, 1, 0]$,
$f_1(a,b) + f_2(a,b) = (2ab + 2(a+b) + 1) + (-2ab - (a+b) - 1) = a+b$;

$$I_1 = -\frac{z_1}{y_1} = -\frac{1}{2}, \qquad \theta_1 = -\frac{y_1}{x_1} = -1,$$

$$I_2 = -\frac{z_2}{y_2} = -1 = \theta_1, \qquad \theta_2 = -\frac{y_2}{x_2} = -\frac{1}{2} = I_1;$$

$$\frac{I_1}{\theta_2} + \frac{I_2}{\theta_1} = \frac{-\frac{1}{2}}{-\frac{1}{2}} + \frac{-1}{-1} = 2.$$

(2) $[x_1, y_1, z_1] + [x_2, y_2, z_2] = [15, 6, 2] + [-15, -5, -2] = [0, 1, 0]$,
$f_1(a,b) + f_2(a,b) = (15ab + 6(a+b) + 2) + (-15ab - 5(a+b) - 2) = a+b$;

$$I_1 = -\frac{z_1}{y_1} = -\frac{1}{3}, \qquad \theta_1 = -\frac{y_1}{x_1} = -\frac{2}{5},$$

$$I_2 = -\frac{z_2}{y_2} = -\frac{2}{5} = \theta_1, \qquad \theta_2 = -\frac{y_2}{x_2} = -\frac{1}{3} = I_1;$$

$$\frac{I_1}{\theta_2} + \frac{I_2}{\theta_1} = \frac{-\frac{1}{3}}{-\frac{1}{3}} + \frac{-\frac{2}{5}}{-\frac{2}{5}} = 2.$$

Example of three rows that sum to a triple with $y^2 - y - xz = 0$ and the rows pairwise sum to a row:

$[1, 7, 42] + [2, 8, 28] + [-3, -14, -70] = [0, 1, 0]$ and

$[1, 7, 42] + [2, 8, 28] = [3, 15, 70]$, another row;

$[1, 7, 42] + [-3, -14, -70] = [-2, -7, -28]$, another row;

$[2, 8, 28] + [-3, -14, -70] = [-1, -6, -42]$, another row.