A general formula for the Frobenius number F(n+1, n+2, .. , n+r)

A positive integer
(1) x = k*(n+1) + j, with k>0 and 0<=j<=n, can be written as a linear combination of n+1, n+2, .. , n+r:
(2) x = c(1)*(n+1) + c(2)*(n+2) + … + c(r)*(n+r) = $\sum_{i=1}^{r} c(i) * (n+i)$
We must solve this equation with suitable coefficients c(i) and look for a condition that these are non-negative.

The transformation (3) x = (n+1)* $\sum_{i=1}^{r} c(i) + \sum_{i=2}^{r} c(i) * (i-1)$
leads to (3a) k = $\sum_{i=1}^{r} c(i)$ (because of j<=n) and
(3b) j = $\sum_{i=2}^{r} c(i) * (i-1)$

Replace c(1) by k − S(j) with S(j) = $\sum_{i=2}^{r} c(i)$, see (3a) (S depends on j, see(5)):
(4) x = $(k - S(j)) * (n+1) + \sum_{i=2}^{r} c(i) * (n+i)$

The coefficients c(i) must be selected such that they satisfy (3b).
We set c(r) = a and, if b>0, c(b+1) = 1 and else c(i) = 0
with a = $\left\lfloor \frac{j}{r-1} \right\rfloor$ and b = j mod (r−1).
Result: $\sum_{i=2}^{r} c(i) * (i-1)$ = (r-1)* $\left\lfloor \frac{j}{r-1} \right\rfloor$ + j mod (r−1) = j.
Thus(3b) is satisfied and (2) is solved.

This leads to S(j) = a if b=0, or S(j) = a+1 otherwise. For both cases:
(5) S(j) = $\left\lceil \frac{j}{r-1} \right\rceil$ and
(6) k ≥ S(n) = $\left\lceil \frac{n}{r-1} \right\rceil$ as j=n must be included.

With this inequality, (2) has been solved such that all coefficients are non-negative.
Thus any x ≥ x_{min} = S(n) * (n+1) has such a linear combination.

On the other hand, (6) is necessary: We have set c(r) to the greatest possible value.
This way, we made S(n) as small as possible because c(r) is the "heaviest" coefficient with the factor r−1.
Thus y = (S(n) -1)* (n+1) + n = x_{min} − 1 is the greatest value of x which does not satisfy the necessary condition (6).

Result: F(n+1, n+2, .. , n+r) = $(n+1) * \left\lceil \frac{n}{r-1} \right\rceil - 1$
= $(n+1) * \text{ceiling}\left(\frac{n}{r-1}\right) - 1, r \geq 2$