July 13, 1970

Dear Neil:

It is nice to have your letter of July 3 with its news of the space packers of the world. I hope Breach got his numbers right in SIAM review (I corrected several but am not sure I got them all). Incidentally I have extended most of his results – doing myself what he refused to do.

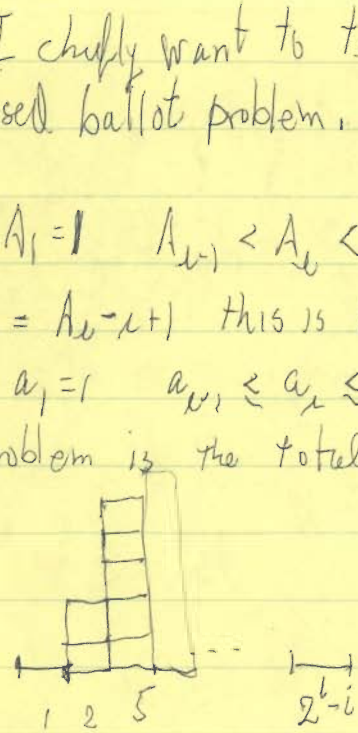
I chiefly want to tell you that Peddicord's problem is a disguised ballot problem. Basically his sequences are defined by

$$A_1 = 1 \qquad A_{i-1} < A_i < 2^i$$

and if $a_i = A_i - i + 1$ this is the same as

$$a_1 = 1 \qquad a_{i-1} \le a_i \le 2^i - i$$

The ballot problem is the total paths with n steps on the lattice



Carlitz, Roselle & Scoville have a paper in the backlog of J.C.T (with ballots unhidden after my editorial remarks), which gives the recurrence for the number of paths when $2^i - i$ is replaced by $f(i)$ as

$$\sum_{j=0} (-1)^j \binom{f(n+1-j)}{j} T(n-j) = 0 \qquad T(0) = 1$$

which is simpler to use that what I worked out for Peddicord.

But also there is the following ~~odd~~ sequence which appears in de Bruijn - Klarners new book (chap. II) ~~and as~~ namely taken

$$2P^*_{2n} = P_{n+1} + P_{2n}$$
$$2P^{*}_{2n+1} = P_n + P_{2n+1}$$

with

$P_0 = P_1 = 1$, $P_n = P_{n-1} + P_{n-2}$ (Fibonacci)

n	0	1	2	3	4	5	6	7	8	9	10	11	12	
P_n	1	1	2	3	5	8	13	21	34	55	89	144	233	45 ✓
P^*_n	1	1	2	2	4	5	9	12	21	30	51	76	127	1224 ✓

It is easy to show ($P_n = P_{n-1} + P_{n-2} = 2P_{n-2} + P_{n-3}$)

$$P^*_{2n} = P^*_{2n-1} + P^*_{2n-2}$$
$$P^*_{2n+1} = P^*_{2n} + P^*_{2n-1} - P_{n-1}$$

Hence

$$P^*(x) = \sum_0^\infty P^*_n x^n = 1 + (x+x^2)P^*(x) - x^3 P(x^2) \qquad P(x) = (1-x-x^2)^{-1}.$$
$$(1-x-x^2)P^*(x) = 1 - x^3 P(x^2) = (1-x^2-x^3-x^4)(1-x^2-x^4)^{-1}$$
$$(1-x-2x^2+x^3+x^5+x^6)P^*(x) = 1-x^2-x^3-x^4$$
$$P^*_n - P^*_{n-1} - 2P^*_{n-2} + P^*_{n-3} + P^*_{n-5} + P^*_{n-6} = \delta_{n0} - \delta_{n2} - \delta_{n3} - \delta_{n4}$$

This is not in your list. References

J. W. Moon, Anoteon "Pattern variants on a square field", Psychometrika **28** (1963), 93-95

Goulomb's book

Yours

John