This sequence can be enumerated using the Polya Enumeration Theorem. We get very straightforwardly for any type of multiset that the desired quantity of all multiset partitions into k sets for some k is given by where $F = f_1 + f_2 + \cdots + f_n$ are the frequencies of the multiset

$$\sum_{k=1}^{F} [A_1^{f_1} A_2^{f_2} \times \cdots \times A_n^{f_n}] Z\left(S_k; -1 + \prod_{q=1}^{n} (1 + A_q)\right).$$

where we refer to the cycle index of the symmetric group. We now use the recurrence by Lovasz for the cycle index $Z(S_k)$ of the multiset operator $\textsc{MSET}_{=k}$ on k slots, which is

$$Z(S_k) = \frac{1}{k} \sum_{\ell=1}^{k} a_\ell Z(S_{k-\ell}) \quad \text{where} \quad Z(S_0) = 1.$$

Next we introduce $T(Q, k)$ where Q is a monomial in the variables A_q and k is a non-negative integer and

$$T(Q, k) = [Q] Z(S_k; -1 + \prod_{A \in Q} (1 + A)).$$

We also put

$$S(Q) = -1 + \prod_{A \in Q} (1 + A).$$

These are the sets that go into the k slots. We thus obtain from the Lovasz recurrence a recurrence for T :

$$T(Q, k) = \frac{1}{k} \sum_{\ell=1}^{k} {\sum_{P \in S(Q)}}' T(Q/P^\ell, k - \ell).$$

The mark on the sum signifies two things, first we only recurse when Q/P^ℓ is a proper monomial including the value one and second, that all monomials are represented by a product

$$A_1^{g_1} A_2^{g_2} \times \cdots \times A_p^{g_p}$$

with the degree sequence $g_1 \le g_2 \le \cdots \le g_p$ in increasing order. This is so that we may properly memoize the recurrence as the result only depends on the partition induced by the monomials (order of variables does not matter). We can take the recurrence for T and more or less translate it directly into CAS code, we just have to take care of the base cases, the most important of which is that when we have reached only one variable then we get a contribution of one if and only if the degree of the variable equals k, and zero otherwise. That is all, if we are after a total count we just add the values for the parameter k.

We can use the following special multisets to compute some example values, where there are n elements in the multiset and m copies of each element. For example, with $n = 7$ elements and $m = 3$ copies of each we have for the possible values starting with $k = 3$ and going to $k = 21$ the sequence

$$1, 714, 84000, 1737813, 11673597, 35162333, 57789691, 59078859, \\ 41165320, 20857585, 8046164, 2441211, 595456, \\ 118300, 19236, 2541, 266, 21, 1.$$

The reader may want to use this to verify their computations. Another example is for $n = 6$ elements and $m = 4$ copies of each we get with k rangeing from $k = 4$ to $k = 24$:

$$1, 201, 18171, 396571, 3053216, 11003801, 22360580, 29114463, \\ 26607981, 18227245, 9816458, 4301588, 1572206, 487670, \\ 129880, 29828, 5901, 995, 140, 15, 1.$$

Another important sanity check is that we should get Stirling numbers of the second kind when $m = 1$ and indeed this check also goes through. The Maple code attached tp the question implements three routines. First a plain enumeration routine that can be used to verify correctness of the more sophisticated alternatives. Second, computation by the Polya Enumeration Theorem directly substituting into the cycle index $Z(S_k)$ and third, the recurrence described above. The number of entries in the memoization table for T is given by the value of the partition function summed for up to the total degree of the inital value of $Q.$