MIXED TREES A335362

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Abstract. We illustrate the layout of mixed trees (trees where a subset of edges is directed/oriented and another subset is undirected) for up to 5 nodes.

1. Nomenclature

Trees are unlabeled simple graphs without cycles. Mixed graphs are graphs where a (possibly empty) subset of the edges is undirected and all others are directed. We count mixed trees by an algorithm that starts from the simple undirected trees, selects a subset of the edges to be oriented, partitions that subset of oriented edges into the two possible orientations, and runs a check on each graph to reduce all these mixed graphs to unique representatives.

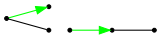
2. 2 nodes

2.1. **2 nodes 0 arcs.**

2.2. **2 nodes 1 arc.**

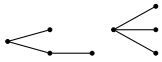
3. 3 nodes

3.1. **3 nodes 0 arcs.** 

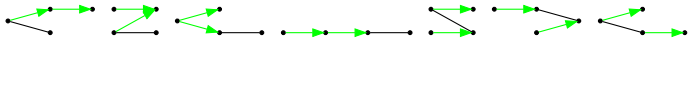
3.2. **3 nodes 1 arc.** 

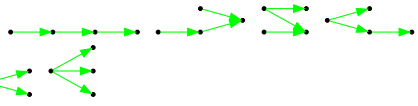
3.3. **3 nodes 2 arcs.** 

4. 4 nodes

4.1. **4 nodes 0 arcs.** 

4.2. **4 nodes 1 arc.** 

4.3. **4 nodes 2 arcs.** 

4.4. **4 nodes 3 arcs.** 

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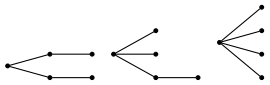
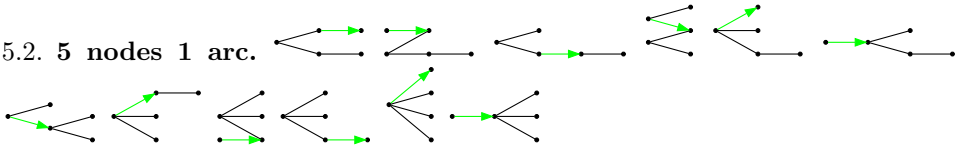
5. 5 nodes

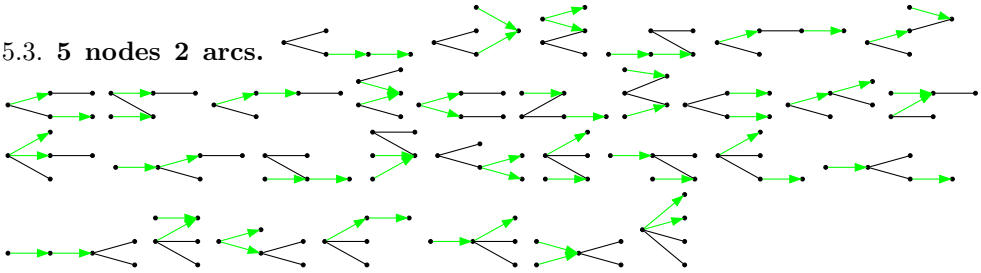
5.1. **5 nodes 0 arcs.**



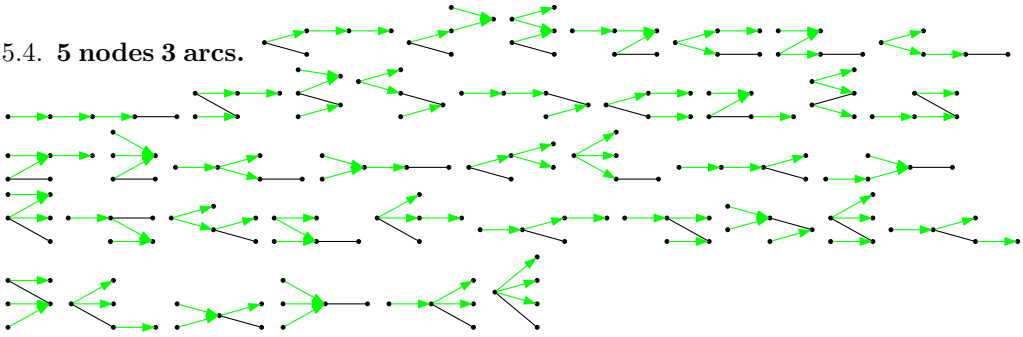
5.2. **5 nodes 1 arc.**



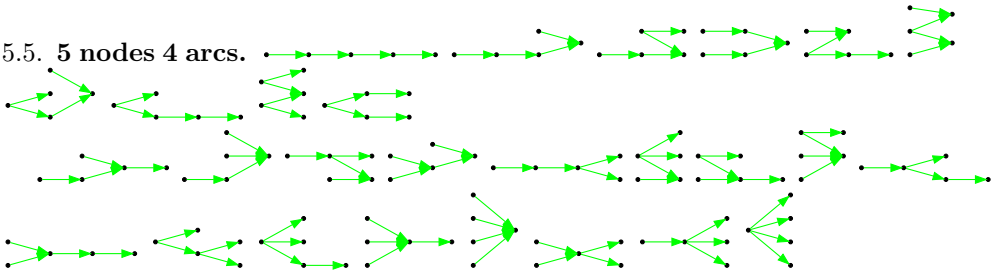
5.3. **5 nodes 2 arcs.**



5.4. **5 nodes 3 arcs.**



5.5. **5 nodes 4 arcs.**



6. Summary

The number of mixed trees on n nodes with d arcs and $n-d-1$ undirected edges is summarized as follows:

$n\backslash d$	0	1	2	3	4	5	6	7	$\sum$
1	1								1
2	1	1							2
3	1	2	3						6
4	2	5	10	8					25
5	3	12	32	40	27				114
6	6	30	99	178	187	91			591
7	11	74	298	692	1019	854	350		3298
8	23	188	890	2538	4751	5692	4074	1376	19532

In the column $d = 0$ we find the number of simple trees [1, A55], and in the diagonal the number of oriented trees [1, A238]. Column $d = 1$ counts the graphs where removing the unique directed edge would split a graph of n nodes into two rooted trees, so this represents [1, A106]. Row sums (as a check) are [1, A6956].

References

1. O. E. I. S. Foundation Inc., *The On-Line Encyclopedia Of Integer Sequences*, (2020), https://oeis.org/. MR 3822822

URL: `http://www.mpia.de/~mathar`

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