Double hyperfactorial

Definition

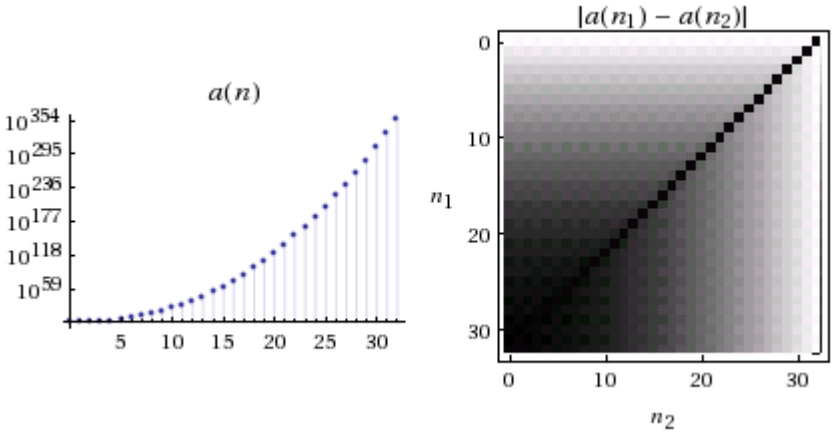
$$H_2(n) = \begin{cases} n^n \cdot (n-2)^{n-2} \cdot \ldots \cdot 5^5 \cdot 3^3 \cdot 1^1, n > 0, n \Rightarrow odd \\ n^n \cdot (n-2)^{n-2} \cdot \ldots \cdot 6^6 \cdot 4^4 \cdot 2^2, n > 0, n \Rightarrow even \\ 0, n = 0 \end{cases}$$

Formula

$$H_2(n) = \prod_{k=0}^{\lfloor \frac{(n-1)}{2} \rfloor} (n-2k)^{n-2k}$$

Recurrence relation

$$a(n) = n^n a(n-2), a(0) = 1, a(1) = 1$$



Properties

$$H_2(n) = \frac{1}{H_2(n-1)} \sqrt{\frac{H_2(2n)}{2^{n(n+1)}}}$$

$$H_2(n) H_2(n-1) = H(n)$$

(see A002109)