# Not Modest

> Hello SeqFans,
> Numbers which, when divided by their last digit, have their first digit as remainder:
>
> 13, 19, 23, 26, 29, 39, 46, 49, 59, 69, 79, 89, 103, 109, 127, ...
>
> Base-10 modest numbers (which are not exactly the same), can be found there: https://oeis.org/A054986
> Best,
> É.

**Alois Heinz** was quick to answer:

> Hello Eric,
>
> 13, 19, 23, 26, 29, 39, 46, 49, 59, 69, 79, 89, 103, 109, 127, 133, 163, 193, 197, 199, 203, 206, 209, 214, 218, 233, 234, 236, 247, 254, 258, 263, 266, 274, 293, 294, 296, 298, 299, 309, 367, 399, 406, 409, 417, 428, 436, 466, 468, 487, 496, 499, 509, 537, 599, 609, 638, 657, 678, 699, 709, 799, 809, 899, 1003, 1009, 1033, 1037, 1063, 1093, 1099, 1107, 1123, 1153, 1177, 1183, 1189, 1213, 1243, 1247, 1273, 1279, 1303, 1317, 1333, 1363, 1369, 1387, 1393, 1423, 1453, 1457, 1459, 1483, 1513, 1527, 1543, 1549, 1573, 1597, ...
>
> Best regards,
> Alois

The next sequence asked would of course have this property:

> Hello SeqFans,
> Numbers which, when divided by their first digit, have their last digit as remainder:
>
> 10, 20, 21, 30, 31, 32, 40, 41, 42, 43, 50, 51, 52, 53, 54, 60, 61,
> 62, 63, 64, 65, 70, 71, 72, 73, 74, 75, 76, 80, 81, 82, 83, 84, 85,
> 86, 87, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, ...
>
> This is not a subsequence of https://oeis.org/A009995 ("Numbers with digits in strictly decreasing order").
>
> Best,
> É.
>

Again, **Alois** was quick:

10, 20, 21, 30, 31, 32, 40, 41, 42, 43, 50, 51, 52, 53, 54,
60, 61, 62, 63, 64, 65, 70, 71, 72, 73, 74, 75, 76, 80, 81,
82, 83, 84, 85, 86, 87, 90, 91, 92, 93, 94, 95, 96, 97, 98,
100, 110, 120, 130, 140, 150, 160, 170, 180, 190, 200, 201,
210, 211, 220, 221, 230, 231, 240, 241, 250, 251, 260, 261,
270, 271, 280, 281, 290, 291, 300, 301, 302, 330, 331, 332,
360, 361, 362, 390, 391, 392, 400, 401, 402, 403, 420, 421,
422, 423, 440, 441, 442, 443, 460, ...

Best regards,
Alois

**Charles Greathouse** adds this comment:

This is an automatic sequence in the terminology of Allouche & Shallit: it can be recognized by a regular expression (or finite-state machine) working on the decimal expansion of the number.

The easy parts:
1.*0
2.*[01]
4.*[02468][0123]
4[0123]
5.*[01234]

8 is routine but somewhat long.

3, 6, and 9 are longer than 8 but not too hard; here's 3:
3([0369]|[147][0369]*[258]|[147][0369]*[147][0369]*[147]|[258][0369]*[147])*[012]

7 is hard. The length of the entire regular expression will be essentially the same as the length of this part, which will be several thousand characters.

**Charles Greathouse**
Analyst/Programmer
Case Western Reserve University

**Maximilian Hasler** has confirmed Alois’s first results and suggested the name “Restricted Modest Numbers” for those integers – good idea!

Many thanks to **Alois**, **Charles** and **Maximilian**!
Best,
É.