OEIS A159867

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Abstract. We illustrate the 1, 3, 12, 60 and 375 free polyedges constructible with 1, 2, 3, 4 and 5 edges embedded in the triangular lattice.

1. Hexagonal Polyedges

We define a set of polyedges consisting of E unit edges of the triangular grid as follows:

- each edge has unit length and connects two nodes of the grid. This implies that at most 6 unit edges may join at a common node;
- pairs of edges with a common node meet at angles of 60, 120 or 180°.
- polyedges are connected, which means one can reach each node of the polyedge starting from any other node traveling along edges.
- polyedges are *free* polyedges in the sense of free polyominoes: translation of a polyedge or operations of the point symmetry group of the regular hexagon (a dihedral group of order 12 with rotations by multiples of 60° or flips along any of the 3 symmetry axes) does not generate a different polyedge.

These polyedges fall roughly into two categories: tree-like (without cycles) or with cycles. (Euler's theorem says the number of edges equals the number of vertices plus the number of rings minus one.) The number of these polyedges seems to be sequence A159867 of the OEIS [2], summarized in Table 1. Explicit illustrations for up to 5 edges are in Figures 1–4.

The polyedges with at least one edge joint at 60° appear in a model of tilings with lozenges [4].

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E	trees	with cycles	total
1	1	0	1
2	3	0	3
3	11	1	12
4	57	3	60
5	347	28	375
6	2372	241	2613
7	16913	2161	19074

Table 1. The number of free connected edge sets with E unit edges.

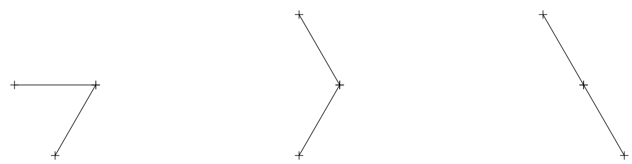

FIGURE 1. The 3 free polyedges with 2 edges.

2. Polyhexes

If a regular hexagon is centered at each node of the edges of the polyedges (such that hexagon edges cut the polyedge edges by half), polyhexes appear [2, A000228]. The number of free polyhexes generated by these means is less than the number of nodes of the polyedges [3, 1, 5]:

(1) In each cycle of the polyedge one of the edges could be removed and still generate the same polyhex; the number of nodes in the polyedge with E edges is $< E+1$ if there are cycles.
(2) The polyedges act as spanning trees for the edge-connected polyhexes, but these spanning trees are not unique as soon as three hexagons meet at a common point. This is correlated with acute 60° joints in the polyedge.
(3) The (reverse) construction of link-incidence diagrams from free polyhexes is 1-to-1, however.

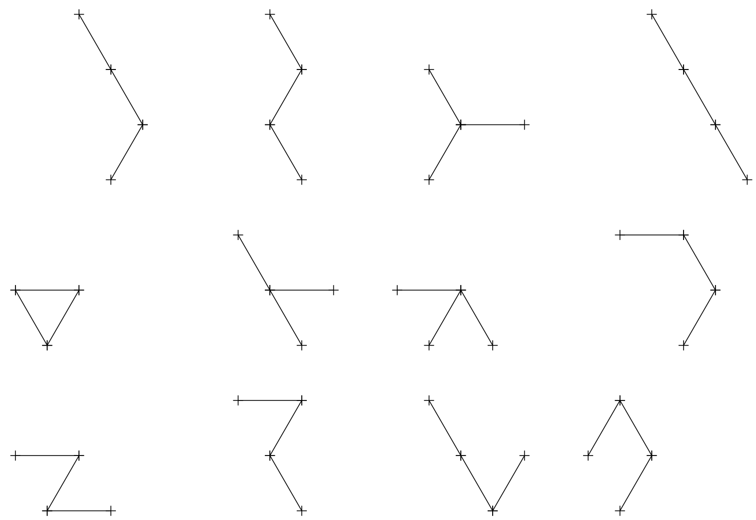

FIGURE 2. The 12 free polyedges with 3 edges. 11 with 4 nodes, 1 with 3 nodes.

REFERENCES

1. Björg N. Cyvin, Jon Brunvoll, and Sven J. Cyvin, *Enumeration of benzenoid systems and other polyhexes*, Topics Current Chemistry **162** (1992), 65–180.
2. O. E. I. S. Foundation Inc., *The On-Line Encyclopedia Of Integer Sequences*, (2020), https://oeis.org/. MR 3822822
3. Frank Harary and Ronald C. Read, *The enumeration of tree-like polyhexes*, Proc. Edin. Math. Soc. **17** (1970), no. 1, 1–13.
4. Richard J. Mathar, *Lozenge tilings of the equilateral triangle*, arXiv:1909.06336 (2019).
5. Nenad Trinajstić, *On the classification of polyhexes*, J. Math. Chem. **9** (1992), 373–380.

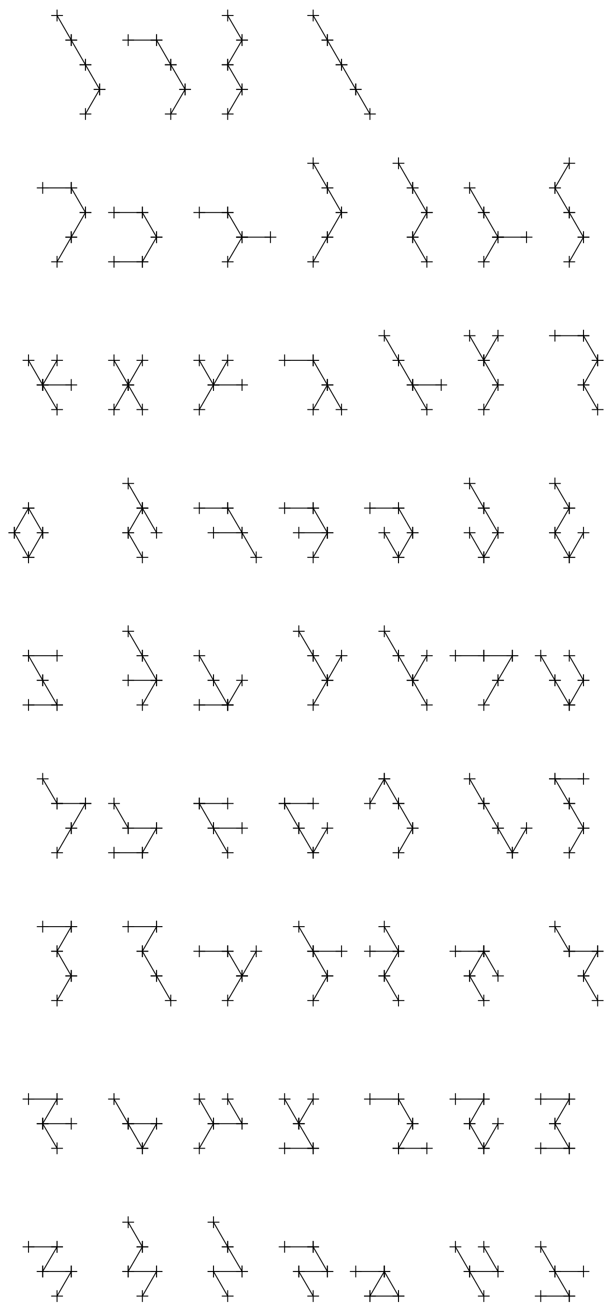

FIGURE 3. The 60 free polyedges with 4 edges. 57 with 5 nodes, 3 with 4 nodes.

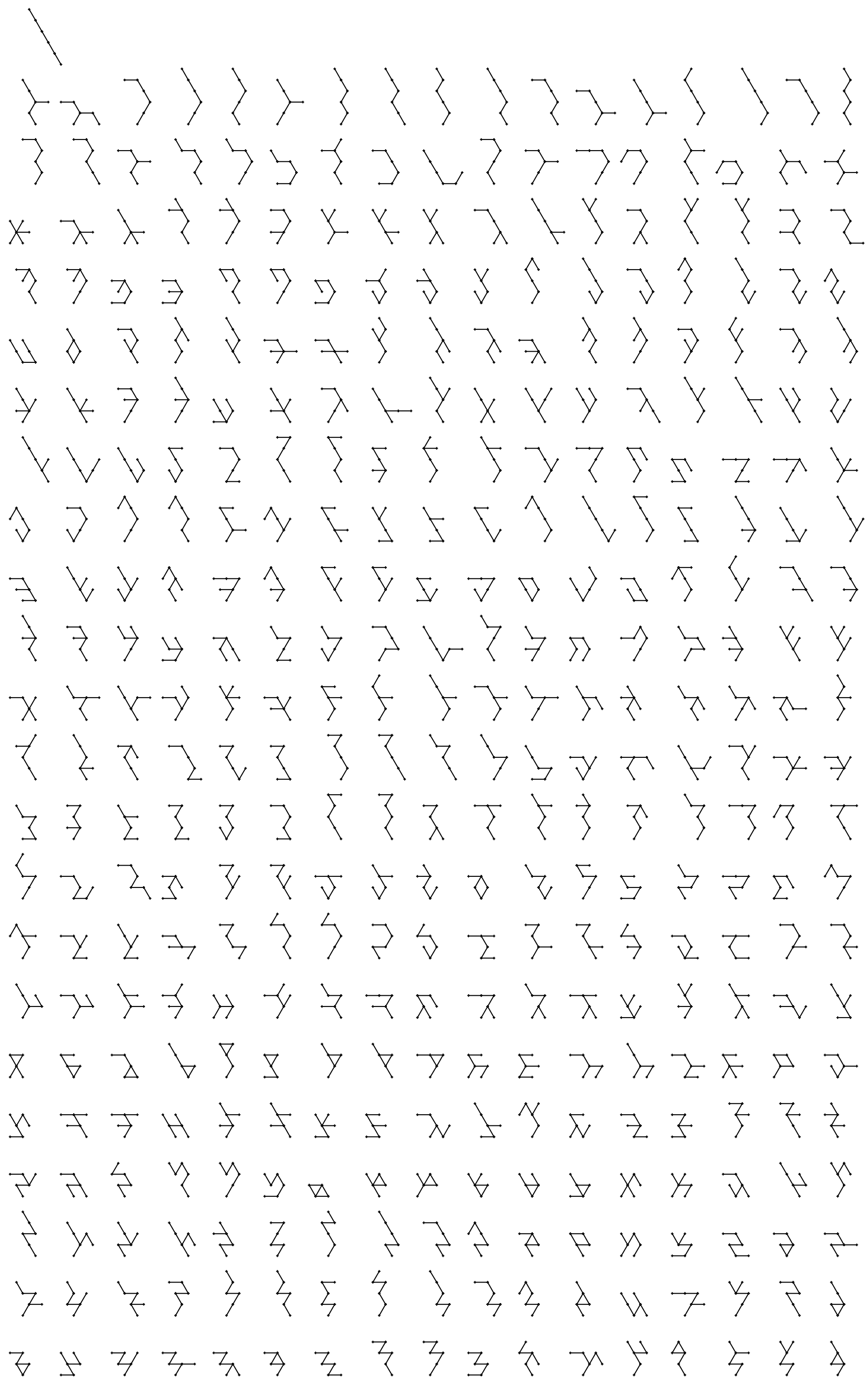

FIGURE 4. The 375 free polyedges with 5 edges. 347 with 6 nodes, 27 with 5 nodes, 1 with 4 nodes.

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