

591

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HERMAN P. ROBINSON

31 DIABLO CIRCLE  
LAFAYETTE, CA 94549  
(415) 283-1861

13 October 1975

Dear Neil:

Fred Gruenberger says he has a sequence of 1000 terms provided by some unknown person a few years ago, and the nature of the sequence is unknown to him. He would like it identified. Do you recognize it? The terms he sent me are

3	4	5	7	10	14	20	29	43
64	95	142	212	317	475	712	1067	1600
2399	3598	5396	8093	12139	18208	27311	40966	61448 ...

The ratios of adjacent terms seem to be approaching  $3/2$ , but I can't match the sequence or ones simply derived from it with anything in your book. Whether the term is even or odd seems somewhat random. I couldn't find a simple recurrence for the series or for its first or second differences.

I've asked Fred for some larger terms, mainly to see how close the ratio of terms gets to  $3/2$ . He says the 1000th term has 177 digits, which is consistent with a ratio of  $3/2$ .

Belated birthday greetings to you.

Sincerely,

*Herman*

Neil:

Your friend's puzzle sequence 3 4 5 7 10 ...

seems to satisfy

$$t_{n+1} = t_n + \left\lfloor \frac{1}{2}(t_n - 1) \right\rfloor$$

Colin

24 x 75

(i.e.  $t_n$  even:  $t_{n+1} = \frac{1}{2}(3t_n - 2)$ )

$t_n$  odd:  $t_{n+1} = \frac{1}{2}(3t_n - 1)$  )

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**Bell Laboratories**

600 Mountain Avenue  
Murray Hill, New Jersey 07974  
Phone (201) 582-3000

November 4, 1975

Mr. H. P. Robinson  
31 Diablo Circle  
Lafayette, California 94549

Dear Herman:

My colleague C. L. Mallows by a great stroke of genius found a recurrence for Fred Gruenberger's sequence

3, 4, 5, 7, 10, 14, 20, 29, 43, 64, 95, 142, . . . .

It is

$$t_{n+1} = t_n + \left[ \frac{1}{2} (t_n - 1) \right],$$

where  $[x]$  denotes the integer part of  $x$ . In other words,

$$t_{n+1} = \frac{1}{2} (3t_n - 2) \text{ if } t_n \text{ is even,}$$

$$t_{n+1} = \frac{1}{2} (3t_n - 1) \text{ if } t_n \text{ is odd.}$$

Best regards,

MH-1216-NJAS=mv

N. J. A. Sloane

← THIS COPY FOR >

Copy to  
Messrs. C. L. Mallows  
Fred Gruenberger