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Gould Publications

HENRY W. GOULD, 1239 COLLEGE AVENUE, MORGANTOWN, W. VA. 26505

12 February 1974

Dr. Neil J. A. Sloane,
Mathematics Research Division,
Bell Laboratories,
600 Mountain Avenue,
Murray Hill, N.J. 07974

Dear Dr. Sloane,

Thanks so much for your most interesting letter of 26 Jan. and the interesting sequence 1,2,3,4,5,6,7,8,9,190,209,48,247,.... with which I have now had a peck of fun around here!

I was able to figure out the right answer in about 3 minutes from the time I opened your letter. I then decided that it would make a rather nice 'intelligence test' to try in our own coffee room over at the University...where we receive staff mail,etc. and where interesting mathematical topics are often raised.

Enclosed is the form in which I circulated the sequence in our WVU staff mailboxes. By the next morning when I returned to my office I found a number of correct (and incorrect) answers. One of my colleagues (working in finite group theory) who knows some number theory claimed he got the (correct) answer in 15 seconds. I found one yellow sheet of paper under my office door with the correct answer (phrased in the precise same words you used!!) and signed by "3 brilliant students who prefer to remain anonymous". All in all, the staff scored well. The biggest question that has been raised, however, is 'How do you know that a next term (with the rule as given) exists at all?' and indeed that is the main question one has to ask about such a sequence.

What do you know about the problem of determining the terms for general n? magnitude? Existence? asymptotic formulas? upper and lower bounds? Is there much in the literature on this one? I don't offhand know of a specific reference.

Glad you liked the cyclic progressive sequence. I have challenged our staff to find this sequence....or any other that offers the feature of changing 1 and only 1 digit when passing from n to n+1.

I have appreciated all your remarks and intend, by the way, to buy a copy of your book to keep in my office, as well as to have our Math.Library buy two copies for our reference shelf.
I have some other sequences in mind arising in the study of chromatic polynomials....more on that in a future communication.

Best regards,

Henry W. Gould

Henry W. Gould

INTELLIGENCE TEST

Give the next term in the sequence

1, 2, 3, 4, 5, 6, 7, 8, 9, 190, 209, 48, 247, 266, 195, 448, ...

and a rule for finding any term.

-- HWG

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COMBINATORIAL RESEARCH INSTITUTE

Prof HENRY W. GOULD, 1239 COLLEGE AVENUE, MORGANTOWN, W. VA. 26505

6 May 1974

Dr. Neil J. A. Sloane,
Bell Laboratories,
Murray Hill, N.J.

Dear Neil,

I have some new sequences to call to your attention that you might have missed and may want to put in a supplement to your book. By the way I enjoyed the first supplement....lots of gold nuggets in that one.

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In the latest MONTHLY, 81(1974),323-343, in his article 'Computer science and its relation to mathematics', Donald E. Knuth notes the sequence 1,1,1,2,4,14,62,.... for the first seven values of a sequence whose general term is"the number of different orders in which a person could change the beads of a necklace from all white to all black, ignoring the operation of rotating and/or flipping the necklace over whenever such an operation preserves the current black/white pattern." He first gives a definition of the sequence using permutations. He calls this the <u>number of necklace permutations</u>. I found a number of entries under necklace in your book, but did not find the sequence. Knuth wonders what the next term is and what is a general formula. Know any more???

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I found another sequence of great interest. Berman and Fryer, in their interesting book 'Introduc.to Combinatorics',p.174, while discussing coloring problems , first show that (as is well known) if <u>6</u> points in the plane, no three collinear, be joined by 15 line segments which form 20 triangles, then if the line segments are colored either red or blue in any way, there will always be a chromatic triangle. In problem 5, same page, if all the diagonals of a <u>66</u>-gon are drawn and all edges colored either red,blue, green, or gold, then there will result a chromatic triangle. Oh, and in problem 2, for a <u>17</u>-gon, if we use red,white,or blue to color the diagonals, a chromatic triangle results. In problem 6, p.175, they ask what is the next number chromatically....and my two students in combinatorics seminar claim to have found that the next number is 327. In general we arrived at the sequence 1,2,3,6,17,66,327, and probably 1958 if the recurrence F(n+1) = (n+1)F(n) - (n-1) is always true. Then we should have 13701, 109602, etc. Do you know any references on this sequence? I could not find it in your tables. There is a certain ambiguity...if you define the sequence to start with 1,2,3,6,...or as <u>1,3,6,17,</u>...or as 1,6,17,.... depending on where you decide to begin a chromatic meaning and not just a formula meaning.

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1,6,17,66,327
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Finally, I enclose a Xerox copy of a rough outline of a proof by my colleague George Trapp (Comp.Science Dept.,WVU) and Franz Delahan (from Kansas) that the n-th term in 1,2,...,8,9,190,209,...always exists, and in fact they do it in an arbitrary base, not just base 10. I conjectured as much. Give me your reaction.

2998

Regards,

Henry

Henry W. Gould, Director

best

[c. MARCH 1974]

Privileged preprint:

By *number* we mean a non-negative integer.

Theorem. Let n be any number and let b be any number greater than 1. Then we can find a number m such that mn can be represented as the sum of n distinct (positive) powers of b.

Corollary: The sequence 1, 2, 3, 4, 5, 6, 7, 8, 9, 190, 209, ... can be continued as far as desired.

Proof of theorem: Fix b and n. Call any number satisfying the property asserted about n in the theorem a *good* number. If there is some number k such that $b^k \equiv 1 \mod v$, then v is said to be a *very good* number.

Lemma 1: Every very good number is a good number.

Lemma 2: Every prime which is not a factor of b is a very good number.

Lemma 3: The product of a good number and a very good number is a good number.

Lemma 4: The product of a good number and a divisor of b is a good number.

The theorem clearly follows from Lemmas 1-4.

Proof of Lemma 1: Suppose v is a very good number and choose k so that $b^k \equiv 1 \bmod v$. Then v divides $\sum_{i=1}^{v} b^{ik}$ and hence v is a good number.

Proof of Lemma 2: Use Fermat's theorem.

Proof of Lemma 3: Let v and g be very good and numbers respectively. Choose k so that $b^k \equiv 1 \bmod v$. Choose distinct positive numbers $j_1, j_2, \ldots j_g$ such that g divides $\sum_{g=1}^{g} b^{j_g}$ and, using Lemma 1, choose distinct positive numbers $l_1, l_2, \ldots, l_v$ such that v divides $\sum_{i=1}^{v} b^{l_i}$. For any non-decreasing sequence of positive numbers $p_1, p_2, \ldots p_v$, v also divides $\sum_{i=1}^{v} b^{l_i + p_i k}$. Choose such p_i so that $j_g + l_i + p_i k$, $g = 1, 2, \ldots g$; $i = 1, 2, \ldots v$ are distinct. Then vg divides $(\sum_{g=1}^{g} b^{j_g})(\sum_{i=1}^{v} b^{l_i + p_i k})$ which is the sum of vg distinct positive powers of b and hence vg is a good number.

Proof of Lemma 4. Let g divide $\sum_{g=1}^{g} b^{j_g}$ as above and suppose t divides b. Choose positive p_i, $i = 1, 2, \ldots, t$ so that $j_g + p_i$, $g = 1, 2, \ldots g$; $i = 1, 2, \ldots, t$ are distinct. Then t divides $\sum_{i=1}^{t} b^{p_i}$ and hence tg divides $(\sum_{g=1}^{g} b^{j_g})(\sum_{i=1}^{t} b^{p_i})$ which consists of tg distinct positive powers of b.

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