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ON A CONJECTURE OF CHOWLA.

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1. Corresponding to every prime $p \geqslant 1$, $p \neq 2, 3$; there is an integer P given by the relation :

$$P = P(p) = \frac{p^2 - 1}{24}.$$

It is easy to prove that excepting P (5), *i.e.*, 1, every $P \equiv 0$, or 2 (mod. 5).

$B = \frac{P-2}{5} = \frac{p^2-49}{120}$

If $P \equiv 0$ (mod. 5), denote P/5 by A, and

If $P \equiv 2$ (mod. 5), denote $(P - 2)/5$ by B.

eg $\frac{17^2-49}{120} = 2$

The A's and the B's < 2000, are listed in section 4.

Under C are listed all those numbers upto 2000, which can be formed by adding together an A and a B.

$\frac{37^2-49}{120}$

I find that every positive integer $i \leqslant 2000$, can be expressed as the sum of an A and a C, and also as the sum of a B and a C.

$\frac{23^2-49}{120} = 4$

This leads me to

Conjecture 1. Every positive integer j which is $\equiv 2$, or 4 (mod. 5), can be expressed as the sum of three P's. If conjecture 1 be true, it shall follow that every positive integer can be expressed as the sum of four P's.

Since all positive integers $\leqslant 23$, can be expressed as the sums of at the most four squares of primes, we readily see that I. Chowla's Conjecture,[1] *viz.*, "Every positive integer can be expressed as the sum of at the most eight squares of primes $p \geqslant 1$;" is contained in Conjecture 1.

2. The following conjecture is noteworthy.

Conjecture 2. Almost all integers which cannot be expressed as the sums of less than four P's are $\equiv 3$ (mod. 5).

If we exclude the use of P (5), *i.e.*, 1, no number of the form $5n + 3$, can be expressed as the sum of less than four P's. Hence only those of the numbers m, of the form $5n + 3$, can be expressed as the sums of less than four P's, for which $(m - 3)/5$ is a C.

Most of the numbers can be expressed as the sums of three A's and also as the sums of three B's. Hence there are only a few numbers of the forms $5n$, and $5n + 1$, which cannot be expressed as the sums of less than four P's. These numbers are listed under D in section 4.

3. In view of Conjecture 1, Chowla's conjecture stands verified[2] for all numbers upto 240000. The following are some of the interesting results which depend upon Conjectures 1 and 2.

(i) Every positive integer of the form $24n + 4$, can be expressed as the sum of four squares of primes.

(ii) Every positive integer of the form $120n + 51$, or $120n + 99$, can be expressed as the sum of three squares of primes.

(iii) The number[3] of positive integers $\leqslant k$, which cannot be expressed as the sums of less than four P's, is $< k/5$.

(iv) The number of positive integers less than k, which cannot be expressed as the sums of less than eight squares of primes, is $< k/20$.

4. The A's < 2000 are :

2381

0, 1, 3, 7, 8, 14, 29, 31, 42, 52, 66, 85, 99, 143, 161, 185, 190, 267, 273, 304, 330, 371, 437, 476, 484, 525, 603, 612, 658, 806, 913, 1015, 1074, 1197, 1261, 1340, 1394, 1463, 1477, 1548, 1606, 1680, 1771, 1912.

The B's < 2000 are :

2382

0, 1, 2, 4, 11, 15, 18, 23, 37, 44, 57, 78, 88, 95, 106, 13[illegible], 156, 205, 221, 232, 249, 310, 323, 414, 429, 452, 550, 576, 639, 667, 715, 785, 816, 837, 946, 1003, 1038, 1122, 1159, 1222, 1313, 15[illegible], 1[illegible]5, 17[illegible]0, 1786, 1817, 1976.

The C's are listed below :

2855

0, 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 14, 15, 16, 18, 19, 21, 22, 23, 24, 25, 26, 29, 30, 31, 32, 33, 35, 37, 38, 40, 42, 43, 44, 45, 46, 47, 49, 51, 52, 53, 54, 56, 57, 58, 60, 63, 64, 65, 66, 67, 68, 70, 7[illegible], 73, 75, 77, 78, 79, 81, 84, 85, 86, 87, 88, 89, 91, 92, 95, 96, 98, 99 ;

1 00, 01, 02, 03, 06, 07, 08, 09, 10, 13, 14, 17, 19, 20, 22, 23, 24, 26, 29, 30, 34, 35, 36, 37, 40, 41, 42, 43, 44, 45, 47, 48, 54, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 70, 72, 73, 76, 77, 79, 80, 84. 85, 86, 87, 89, 90, 91, 92, 94, 96, 98 ;

2 00, 01, 03, 05, 06, 08, 12, 13, 18, 19, 21, 22, 24, 27, 28, 29, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 46, 47, 49, 50, 52, 55, 56, 57, 61, [illegible], 67, 68, 69, 71, 73, 74, 75, 77, 78, 80, 82, 84, 85, 87, 88, 90, 91, 95, 96, 9[illegible], 99 ;

3 01, 04, 05, 06, 08, 10, 11, 13, 15, 17, 18, 19, 20, 22, 23, 24, 26, 27, 30, 31, 32, 34, 37, 39, 41, 45, 46, 48, 51, 52, 53, 54, 55, 61, 62, 64, 65, 66, 67, 68, 71, 72, 73, 74, 75, 76, 79, 82, 86, 87, 89, 90, 92, 93, 94, 95, 99 ;

4 01, 06, 07, 08, 09, 10, 11, 14, 15, 17, 18, 21, 22, 23, 25, 28, 29, 30, 32, 34, 36, 37, 38, 39, 41, 43, 45, 48, 49, 52, 53, 55, 56, 58, 59, 60, 64, 66, 71, 72, 74, 76, 77, 78, 80, 81, 83, 84, 85, 86, 87, 88, 91, 94, 95, 99 ;

5 00, 02, 04, 05, 07, 08, 09, 13, 14, 15, 16, 18, 20, 21, 22, 25, 26, 27, 28, 29, 32, 33, 35, 36, 37, 40, 41, 43, 48, 50, 51, 53, 54, 57, 58, 62, 64, 69, 71, 72, 75, 76, 77, 79, 81, 82, 83, 84, 90, 92, 93, 95, 96, 99 ;

6 02, 03, 04, 05, 07, 10, 12, 13, 14, 16, 18, 19, 20, 21, 23, 26, 27, 28, 30, 31, 32, 35, 37, 39, 40, 42, 46, 47, 49, 53, 56, 58, 59, 60, 61, 62, 67, 68, 69, 70, 73, 74, 75, 76, 81, 86, 87, 89, 90, 91, 93, 94, 95, 96, 97, 98 ;

7 00, 02, 05, 07, 08, 09, 11, 15, 16, 18, 19, 22, 23, 24, 25, 29, 30, 33, 35, 36, 37, 38, 40, 44, 46, 47, 52, 53, 56, 57, 59, 60, 61, 64, 66, 67, 68, 74, 81, 82, 85, 86, 88, 92, 93, 94, 99 ;

8 00, 06, 07, 08, 10, 14, 16, 17, 19, 21, 23, 24, 27, 28, 29, 30, 33, 35, 37, 38, 40, 43, 44, 45, 47, 48, 49, 50, 51, 52, 54, 57, 58, 61, 63, 66, 68, 70, 76, 79, 80, 82, 84, 89, 90, 94, 98 ;

9 00, 01, 03, 05, 06, 07, 12, 13, 14, 15, 17, 21, 22, 24, 26, 28, 31, 34, 35, 36, 39, 40, 43, 46, 47, 49, 50, 53, 54, 57, 59, 60, 62, 68, 69, 70, 71, 75, 77, 80, 81, 82, 87, 88, 91, 97, 98 ;

10 01, 03, 04, 06, 08, 10, 11, 12, 13, 15, 16, 17, 19, 22, 26, 27, 30, 31, 32, 33, 34, 38, 39, 41, 45, 46, 47, 52, 55, 58, 59, 60, 64, 67, 69, 72, 74, 75, 76, 78, 80, 83, 85, 86, 87, 88, 89, 90, 92, 93, 97 ;

11 01, 02, 03, 04, 07, 10, 11, 15, 16, 18, 20, 21, 22, 23, 25, 29, 30, 31, 34, 36, 37, 41, 43, 45, 46, 49, 51, 52, 53, 56, 59, 60, 62, 64, 66, 67, 69, 71, 73, 74, 79, 80, 81, 87, 88, 90, 91, 92, 93, 97, 98, 99 ;

12 01, 07, 08, 11, 12, 13, 15, 19, 20, 21, 22, 23, 25, 28, 29, 30, 34, 35, 36, 40, 41, 42, 44, 47, 50, 51, 53, 54, 58, 61, 62, 63, 64, 65, 69, 70, 72, 74, 75, 76, 79, 83, 84, 85, 88, 92, 95, 97, 98 ;

13 00, 02, 03, 05, 06, 07, 10, 11, 12, 13, 14, 16, 17, 18, 20, 21, 23, 25, 27, 31, 33, 38, 39, 40, 41, 42, 44, 49, 51, 53, 55, 56, 58, 62, 63, 65, 67, 68, 73, 74, 77, 79, 82, 83, 84, 88, 89, 94, 95, 96, 97, 98 ;

14 02, 05, 07, 09, 12, 17, 18, 19, 22, 26, 28, 29, 30, 31, 32, 35, 38, 40, 43, 44, 45, 46, 49, 51, 52, 56, 63, 64, 65, 66, 67, 71, 72, 73, 74, 75, 77, 78, 79, 81, 82, 86, 87, 88, 89, 92, 93, 95, 96, 98 ;

15 00, 03, 07, 10, 14, 20, 21, 22, 26, 28, 30, 34, 41, 45, 48, 49, 50, 51, 52, 55, 58, 59, 61, 62, 63, 65, 66, 69, 70, 71, 72, 76, 80, 83, 84, 85, 86, 89, 91, 92, 93, 96, 97, 98, 99;

16 04, 05, 06, 07, 08, 10, 11, 14, 15, 17, 19, 21, 22, 24, 26, 28, 29, 33, 35, 36, 38, 41, 42, 43, 47, 49, 50, 54, 59, 61, 63, 64, 66, 68, 75, 77, 80, 81, 82, 84, 87, 90, 91, 94, 95, 96, 98;

17 01, 03, 04, 05, 06, 09, 12, 13, 17, 20, 23, 24, 25, 26, 29, 30, 34, 37, 40, 41, 43, 47, 48, 50, 52, 53, 54, 58, 62, 68, 69, 71, 72, 73, 75, 78, 80, 82, 86, 87, 89, 92, 93, 94, 96, 97;

18 00, 06, 08, 09, 11, 14, 15, 17, 18, 20, 23, 24, 25, 27, 28, 29, 31, 34, 35, 36, 37, 38, 39, 44, 46, 48, 49, 52, 55, 58, 59, 64, 66, 69, 71, 77, 80, 83, 85, 90, 91, 92;

19 00, 01, 02, 05, 06, 08, 11, 12, 13, 14, 15, 16, 23, 25, 27, 28, 29, 30, 33, 35, 39, 44, 47, 49, 51, 56, 60, 61, 62, 65, 69, 70, 71, 76, 77, 78, 79, 82, 83, 84, 90, 92, 99;

20 00.

The following are the D's:

1125, 1310, 1360, 1936, 1971, 2245, 3895, 5000, 5055, 5066, 6270, 6501, 6540, 6920, 8055.

REFERENCES.

1. I. Chowla, *Proc. Ind. Acad. Sci.*, 1935, 1, 451–453.
2. H. Gupta, *Proc. Ind. Acad. Sci.*, 1935, 1, 789–794.
3. For $k = 10000$, this is > 1000.

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